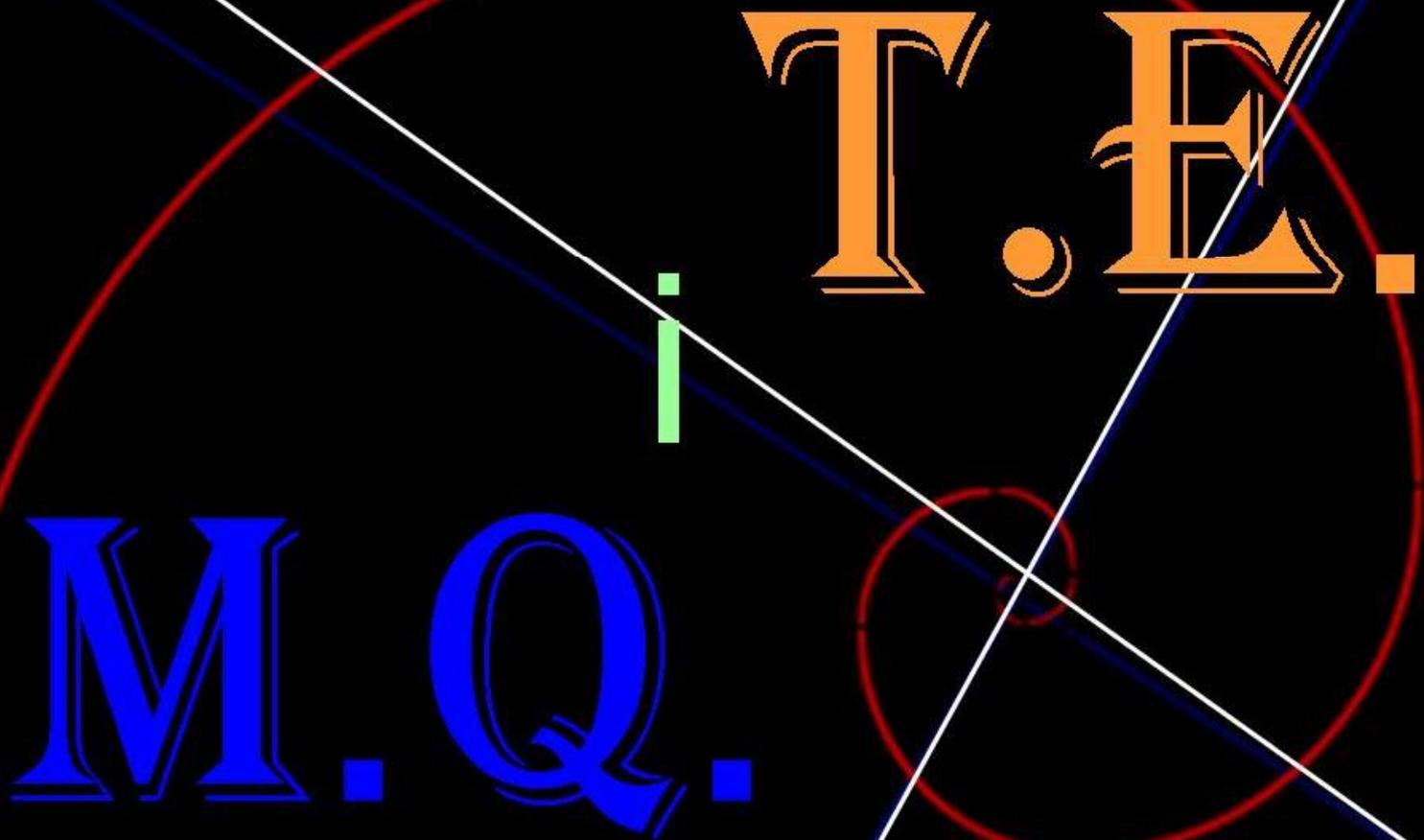


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Technology Diffusion and its Effects on Social Inequalities

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Technology Diffusion and Its Effects on Social Inequalities

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Abstract

We develop a dynamic general-equilibrium model in which growth is driven by a skill-biased technology *diffusion* to reproduce trends in the income inequality, and the labor and skills supplies of the United States between 1969 and 1996. We incorporate education and leisure–labor decisions, and human-capital accumulation. We provide an explanation to why individuals invest in human capital when the investment premium is going down and why the skill-premium is going up when the skills supply is increasing. In addition, our model is the first general-equilibrium model, to our knowledge, that is consistent with a decline of unskilled wages and low growth of productivity in which the effects of a skill-biased technology diffusion on social inequalities are studied. We show that the effects of labor supply decisions on the skill premium cannot be neglected in a diffusion model.

JEL classification: J22, J23, J24, J31.

Keywords: Heterogeneous agents, Inequality, Skill-Biased Technical Change.

1 Introduction

The income inequality has increased since the 1970s in a large number of OECD countries (Gottschalk and Smeeding, 2000). This increase is mainly caused by a rise in the wage differential between and within groups (Autor and Kerarney, 2006). For example, in the USA the skill premium (SP), which is the wage of skilled relative to unskilled labor, has grown in spite of a sharp increase in the relative supply of skilled individuals in the last thirty years (Figure 1). The skill-biased technology change (SBTC) has been one of the most accepted explanations for the skill premium evolution (Aghion, 2002; Acemoglu, 2002). Yet, the supply-side effects have received little attention in the SBTC literature. However, Katz and Murphy (1992) show that observed fluctuations in the growth rate of the relative skills supply combined with smooth trend demand growth in favor of more highly educated workers can explain movements in the college premium over the 1963-1987. Indeed, the authors demonstrate that the pattern of changes observed in the skill-premium over 1963-1987 are strongly correlated to the differences across the three decades in the growth rate of the relative skills supply. For example, the largest increase in the supply of skills observed during the 1971-1979 period, period in which the skill-premium declines, and the smallest increase occurs during 1979-1987 period, period in which the skill-premium

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increased sharply. Given the empirical evidence, we claim that the relative skills supply is determinant to explain the skill-premium evolution. Additionally, once the technology diffusion implies changes on expected returns of education, we also claim that the skills supply is determined by this technology diffusion.

In this paper we study how the diffusion of a SBTC affects the households' labor supply and their investment in education when credit markets are absent. Households' choices are analysed in the context of a general equilibrium model in which households allocate their time to production and education activities. Households differ in their education levels and abilities to acquire skills. We also look at the role such decisions play in the evolution of the skill-premium (SP) and the consumption inequality over the technology diffusion cycle. The effects of an SBTC diffusion on the leisure–labor decisions and education investments have not been analyzed previously, in particular, during the technology diffusion cycle.

Since the SBTC requires human capital as an input, and the agents are forward-looking, we ask whether a technology diffusion is able to explain both the SP and skills supply evolution. We focus on the differences between workers before and after entering the labor market, accounting for pre-labor market investments (schooling) and job market investments (training). The model is calibrated to the US economy in 1969 when the economy is assumed to have been in transition between two types of technologies: the (new) skill-biased technology and the (old) existent technology.

Motivated by the lack of attention given to the supply-side effects of an SBTC diffusion, we extend the [Aghion \(2002\)](#) model of diffusion. First, we introduce optimal individual labor supply, the analysis of which is essential due to non-constant lifetime earnings associated with the technology diffusion cycle. Second, agents have different initial and lifetime skills evolutions: unskilled agents optimize their investment in education in order to obtain the necessary skills, whereas skilled agents are skilled over their entire lifetime. The accumulation of skills requires of course time and agents abdicate from their leisure or production time in order to invest in education. Based on these extensions we build a dynamic general-equilibrium model with a SBTC diffusion together with education and leisure–labor decisions based on expected earnings of different skilled occupations.

The model has two main features. First, there is the appearance of a new technology that is more productive and skill-biased; this technology demands skilled workers and diffuses throughout all sectors of the economy. It requires a period of experimentation and adaptation, which initially delays its diffusion. Afterwards we observe the diffusion speed-up due to social learning effects. Second, when this technology diffusion starts there is an ex-ante heterogeneity of individuals. A proportion of individuals already have the skills required by the new technology. The remaining individuals might acquire these skills by investing in schooling activities, which obviously comes at a (disutility) cost. Individuals have forward-looking expectations and allocate their time between production and school, according to their expected future wage differential. The productivity of schooling activities is smaller than one, which implies that it is not instantaneous.

The general equilibrium allows us to examine not only the traditional effects of technology diffusion on the income inequality but also its effects on the supply of skilled and unskilled labor as well as on education. As technology diffusion is a non-stationary process, the effects of labor and education are different in each of the diffusion phases.

The model is of course highly stylized, and we are mainly concerned with the overall trends. Moreover, the results discussed here are merely the means to understand the effects of technology *diffusion* in a general-equilibrium framework that includes leisure–labor and education decisions.

Findings of this paper can be summarized as follows. First, we find that the skill-biased technology diffusion, together with the endogenous human capital accumulation hypothesis, reproduces the trends observed for both the skills supply and skill premium from 1969 to 1996. The two driving forces are a firms' learning process and individuals' expectations on education returns. Initially the growth rate of the demand is small because firms need to experiment and learn to use new technology. In contrast, the growth rate of supply of skills is relatively high because it is when individuals maximize their returns to investments in education. After this initial period, the learning process accelerates and expected returns of education start to decrease. The combination of these two forces lead to a decline of the skill-premium in the 70s and a strong increase of the skill-premium in the 80s. Thus, the first finding arises from non-constant¹ inter-temporal effects, being these effects stronger in the 70's than in the 80's, because of the higher profitability of investments in education in the 70's.

Second, we find that the supply of skilled labor decreases throughout the technology diffusion cycle at a non constant rate. As wages of skilled workers increase over time, with the exception of the 70's, the decrease of working hours by skilled individuals illustrates the dominance of the income effect from the 80's. Additionally, our model results show that the skilled individuals work more hours than the unskilled, in the last twenty-five years of their lifetime. These facts were also found in the data as documented by [Erosa et al. \(2009\)](#) and [Heathcote et al. \(2010\)](#). [Erosa et al. \(2009\)](#) justify these facts as a consequence of non-linear wages over time and heterogeneous agents, being these two features captured by our model. In our model, the non-linearity of wages is due to the technology diffusion process and the heterogeneity is due to the existence of different skills. Regarding the unskilled workers, we observe an increase of the unskilled labor supply over time. This rise also illustrates a dominance of the income effect from the 80's, period in which unskilled wages start to decline.

Third, our model reproduces both intra- and inter-temporal consumption inequality between educational groups, as found in [Attanazio \(1999\)](#) and [Krueger and Perri \(2006\)](#). The consumption of skilled individuals is greater than the consumption of unskilled individuals. Moreover, this consumption inequality tends to increase over time for two reasons. First, the skill-premium trend is positive, reflecting an increase of wages of skilled individuals relative to unskilled ones. Second, there is a dominance of the substitution effect meaning that this rise of the skill-premium is accompanied by an increase of working hours of skilled individuals relative to unskilled. Finally, we replicate a decline of the unskilled wages over time, due in particular to a reduction of the demand for unskilled workers. Our results are consistent with the low growth of productivity, because the inequality arises from the diffusion of a skill-biased technology rather than constant increases of productivity growth.

The paper is organized as follows. In Section 2 we present a brief survey of the problem illustrated so far and discuss a few prominent proposed solutions. In Section 3 we describe the optimization problem of companies in a monopolistic competition with two technologies, one being more productive than the other one. In Section 4 we outline the consumer household problem and the general-equilibrium model for the economy with leisure-labor decisions and an endogenous evolution of skills through the accumulation of human capital. In Section 5 we present the numerical results and in Section 6 we conclude. Information on the algorithms used can be found in [A.1](#).

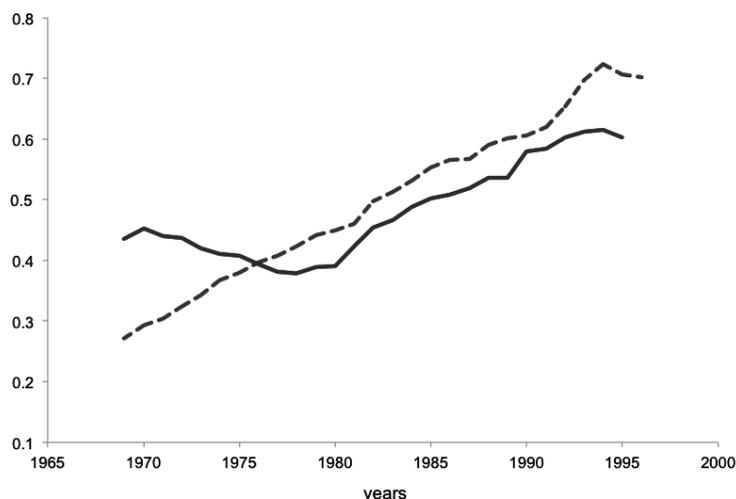


Figure 1: The behavior of the logarithm of the college premium and relative skills supply. Adapted from: Acemoglu (2002). College wage premium: solid line. Relative supply of skills: dashed line.

2 The Problem, Its Solutions, and Their Problems

The puzzle in Figure 1 is why individuals invest in human capital when the premium on this investment is going down and why the skill-premium is increasing when the relative skills supply of educated individuals is going up. So far the literature has only looked at the latter and several explanations have been provided that go from trade theory to SBTC theory².

A first explanation is associated with changes in immigration flows. In the 80's there was an increase of immigration of low-skilled workers into the US. Thus the influx of workers reduced the wage of other low-skill workers, increasing the skill-premium (Borjas et al., 1996). These effects on the skill-premium were reinforced by globalization. According to the Heckscher–Ohlin theory, a second explanation, globalization is deemed responsible for the rise in the demand of skilled labor in the developed nations, where skilled labor is cheaper than in developing countries, and, therefore, the rise of the skill-premium. A third explanation pertains to a rapid skill-biased technology change³ (SBTC), a sudden shift in the technology production that favors skilled labor, and the hypothesis that capital and skills are complementary. Krusell et al. (2000), using a neoclassical framework⁴ argue that the complementarity of skilled labor and physical equipment – contrary to unskilled labor – is the main reason for the increase in the SP. This argument implies that the inequality should be followed by a rise in the productivity growth, which was much slower after 1975 than during the previous two decades. However, the main debate is about two competing Schumpeterian growth mechanisms.

The first Schumpeterian mechanism (Acemoglu, 1998, 2002) is the pure technology progress model. It emphasizes the interplay between the supply of skilled labor and the endogenous rate of innovation. The increase in the labor supply is basically the engine of innovation, and consequently the skill

¹Note that our model is non-stationary and there is imperfect competition. Thus, both inter- and intra-temporal effects are not constant over time.

²See Aghion and Howitt (2002) for a review of this literature.

³Autor and Krueger (1998), Berman and Griliches (1994) and Murphy and Welch (1992) have provided evidence in favor of the SBTC.

⁴In Krusell et al. (2000) the SBTC reflects the growth of the equipments' stock and by hypothesis capital and skills are complements in the aggregate production function. Krusell et al. consider a three-factor production function as Stokey and Lucas (1996), in which the elasticity of substitution of capital equipment and unskilled labor is higher than that between capital equipment and skilled labor.

premium rise. [Acemoglu \(1998, 2002\)](#) supposes that the final output is produced by two types of intermediate products—one that requires college graduates to be produced (skill-intensive product) and one that can be produced by high-school graduates. Acemoglu argues that the increase in the relative supply of college-educated workers in the 1970s directed the technology change, during which it became more skill-biased than before because of a ‘market size’ effect. In the short run, the increase in skilled college laborers reduces their relative wages (i.e. a slowdown in the SP), which leads to two opposing effects on the relative profitability of skill-biased products. On the one hand, the decrease of the SP reduces the relative price of skill-biased products, and consequently the relative cost-per-unit that can be saved by quality improvements. This is the substitution effect, which causes the economy to move along a downward-sloping relative demand curve. On the other hand, the increase in the skills supply implies that in equilibrium there is a relative increase in the quantity of skill-intensive products, so the per-unit gain of a quality improvement increases. This is the ‘market size’ effect, also known as the directed-technology effect, which dominates when the elasticity of substitution between the two kinds of intermediate goods in the final-goods sector is large.

The pure technology progress models fall short of a satisfactory explanation of the issues involved. First, they do not offer an explanation as to why the wage inequality decreased during other historical episodes of sustained increases in the supply of educated laborers. Indeed, [Goldin and Katz \(1999\)](#) and [Katz and Autor \(1999\)](#) show that despite a pronounced increase in the skilled-labor supply between 1915 and 1950, following the high-school movement, the relative wages of college-educated workers fell continuously during the first half of the century. Second, pure technology progress models take as given the overall level of innovation and the increase in the relative supply of skills necessary to speed up the SBTC, attributing to the increase in the relative supply of skills to the post-war baby boom. Still, one can only but wonder why the individuals specifically chose to invest in college education. Third, empirical evidence contradicts the R&D-based models of growth that predict that the innovation rate increases with the supply of skills. Fourth, the model provided by [Acemoglu \(2002\)](#) only generates an increasing SP if the substitutability between skilled and unskilled workers is greater than 2, which some empirical studies have shown to be around 1.67 ([Krusell et al., 2000](#)). Finally, pure technology progress models are unable to replicate the reduction of unskilled wages over time.

The second Schumpeterian mechanism is based on the technology diffusion process ([Aghion, 1998, 2002](#)). Mainly, it states the idea that a new technology diffuses through industrialized economies. This second mechanism reconciles market-size theory with the empirical evidence. It assumes that there is a long learning period before technology becomes effective. According to this view, the aggregate productivity slowdown is due to the time necessary to produce and develop the new technological paradigm, generally called the general-purpose technology (GPT). Thus, there is a *diffusion* of a new skill-biased technology across all sectors of the economy as the source of both the income inequality and the skills supply evolution. The diffusion of an SBTC increases the skill premium via changes in the demand for skills. This alternative explanation is also compatible with the low growth of the productivity observed in the USA, given that it is not dependent on a uniformly increasing innovation growth rate, as in the pure technology progress models.

[Greenwood and Yorukoglu \(1997\)](#) point out that the diffusion of a new GPT in the field of information and communication technology (ICT) increases the demand for skills when the new GPT diffuses through the economy because experimentation and adoption require more skilled labor, which accounts for the slowdown and increase in the SP. At the same time, the adoption, diffusion, learning and improvement costs justify the slow productivity growth. [Howitt \(1998\)](#) argues that induced obsolescence is another channel through which the introduction of a new GPT can cause a long-lasting slowdown

in the observed growth rate. [Aghion \(2002\)](#) shows, in a *partial-equilibrium* framework, that the technology diffusion model provides an important explanation for the connection between the slowdown of and the increase in the SP. However, the continued increase in the relative supply of skills remains unexplained.

In this study, we extend the [Aghion \(2002\)](#) model introducing human-capital accumulation through investments in education and analyze the general equilibrium implications. These extensions provide an explanation for the continuous increase in the relative supply of skills compatible with the skill premium path for the US from 1969 to 1996. Assuming discrete schooling choice we show that the SBTC diffusion assumption allows us to replicate the SP trends as well as the relative skills supply evolution.

The impact of the SBTC diffusion is examined on the households' labor supply decisions and investments in education. The labor supply decisions due to population changes are shown to have dramatic effects on growth and business cycles. Therefore, the effect of labor supply decisions on the SP cannot be neglected in a diffusion model.

3 Model

Here we extend the [Aghion \(2002\)](#) cyclical-growth model. We introduce heterogeneous⁵ agents and analyse the effects of leisure-labor decisions on consumption, human-capital accumulation and the income inequality. Furthermore, we construct the dynamic transition path for a neo-Schumpeterian dynamic general-equilibrium model without limiting ourselves to steady-state analyses.

3.1 Population

A uniformly distributed heterogeneous population of size L consists of two subpopulations, $L = i^u \cup i^s$. The labels 's' and 'u' designate the skilled and unskilled subpopulations respectively, and i denotes individuals. Skilled individuals are individuals with a college degree while unskilled individuals are individuals without a college degree. A degree gives individuals the skills they need to work with the new GPT. Let L_t^s and L_t^u denote the measure of the sets of skilled and unskilled workers at time t . The population size L is constant and normalized to $L = 1$. The composition of the population changes over time and, therefore, L_t^s and L_t^u are time-dependent. Each individual has a finite lifetime of T periods, and is endowed with one unit of labor.

3.2 Firms

The fundamental principle of technology diffusion models is that a new GPT does not come ready to use; it requires a new set of intermediate goods, and the economy must wait until a critical mass of intermediate components has been reached. Once it is profitable to switch, firms move to the new GPT. The diffusion of a new GPT thus consists of a wave of secondary innovations creating new and improved products or processes in a specific sector, which are adaptations of the same GPT from other sectors. At period t there is a continuum of firms j , distributed uniformly on the unit interval. Of these

⁵The heterogeneity is restricted to two types, skilled and unskilled, making the model more tractable and compatible with the largest transition observed, which is from high-school-educated to college-educated workers. In the USA, the largest transition in the tax brackets also occurs for these groups.

firms, there are the ones labeled ‘new’ and the ones labeled ‘old’; these are the firms using the new and old GPTs respectively.

Let n_t denote the proportion of firms that are already applying the new GPT, and hence $(1 - n_t)$ is the proportion of firms that are still using the old GPT. The firms applying the new GPT employ skilled workers, and the firms applying the old GPT employ both skilled and unskilled workers. As the GPT permeates the economy, the demand for skilled labor increases. The diffusion may take some time to set in, but once it starts, it tends to pervade over a relatively short time. Afterwards there is an acceleration in the diffusion, and consequently in the demand for skilled labor as well.

A firm to move from the old to the new GPT must acquire a ‘template’, experiment, and succeed. The firms’ transition from the old to the new GPT is given by an exogenous Poisson distribution. The reason that the diffusion of a new technology follows a nonlinear and logistic path has to do with the complementarity, or network externalities between sectors, and social learning effects.

Complementarity is due to some exogenous factors that may delay the transition to the new GPT, such as the real labor costs, trade liberalization, or the intensification of the product market competition.

Social learning effects are related to learning processes; firms learn to use the new technology not only because they are using it, but also from the experience gathered in other sectors where the technology has already been applied successfully. This so-called threshold effect in cross-sector imitation (Aghion, 2002) implies that the larger the proportion of firms that have already implemented the new GPT, the more readily the knowledge and experience is available, and, therefore, the greater the benefits to firms that intend to use new technology. This snowball effect accelerates the diffusion, and thus the demand for skilled labor, which in turn increases the skill premium.

Assumption 1. *The evolution of n_t is given by the relation*

$$n_t = e^{-\lambda} \sum_{k=0}^t \frac{\lambda^k}{k!}, \quad (1)$$

where λ is the arrival rate of the template for the new GPT, and $n_0 > 0$.

Equation (1) follows a cumulative Poisson distribution, as $e^{-\lambda} \lambda^k / k!$ is the associated probability mass function.

3.3 Market Clearing Conditions

Whenever a new technology is adopted, there are initially too few firms using it to absorb the entire skilled labor force. Therefore, the demand for skilled workers is too low and skilled workers supply labor to firms using old technology and firms using the new technology. This means the labor market is not segmented. The labor market is not segmented when the equilibrium wage is unique. In this case some skilled workers are employed by the old firms and both type of workers received the same wage (w).

After the initial phase the number of sectors using new technology is already large enough to take up all the skilled workers, and thus the labor market becomes segmented⁶. Afterwards, skilled workers

⁶To verify if the labor market is segmented, we solve the optimization assuming that the labor market is segmented, i.e., using the labor market condition for segmented markets. Then, if $w^s \geq w^u$ the market is segmented. Otherwise, if $w^s \leq w^u$ the labor market is unsegmented; and we solve the optimization problem again, but now we use the labor market condition for unsegmented markets.

only supply labor to firms using the new GPT, because the new GPT is more productive; once the labor market becomes segmented skilled workers' wages are higher than unskilled workers' wages, $w^s \geq w^u$. Note that unskilled workers can only work in the sectors using the old because they do not have the skills needed to work with the new GPT. Thus we have two labor markets: the unskilled and the skilled labor market.

Markets are not segmented at the beginning of the implementation of new technology, but they become endogenously separated as the technology diffuses. The segmentation is due to the type of relationship between the demand for and supply of skills. As soon as the demand for skilled workers is higher than the skills supply, that is as soon as $w^u < w^s$, the labor market becomes segmented. Because in our model the skills and labor supply are endogenously determined, to check the time in which the labor market becomes segmented we solve the households' optimization problem considering the labor market segmentation hypothesis and if $w^u < w^s$, it confirms that the labor market is segmented. Otherwise, we resolve again the optimization problem, now considering the unsegmented labor market hypothesis and keep these results as solution to the households' problem. Moreover, as we assume equilibrium in the labor market, that is the demand equals the labor supply, both economies—segmented and non-segmented—are in equilibrium.

When the labor market is still unsegmented, there is only one labor market clearing condition:

$$L_t^h = \int l_t(i)L(i) di = \int x_t(j) dj = (1 - n_t)x_t(j)^{\text{old}} + n_t x_t(j)^{\text{new}}, \quad (2)$$

where $0 < l_t(i) < 1$ is the number of hours of work supplied by either class of workers, and $x_t^{\text{new}}(j)$ and $x_t^{\text{old}}(j)$ denote the labor demands by firms using new technology and old, respectively.

As firms using the new GPT are more productive than the ones using the old GPT, when the labor market is segmented the skilled workers are employed at higher wages, that is $w^s > w^u$, where w^s denotes the real wage of skilled workers, and w^u the real wage of unskilled workers. Once the labor market has become segmented, there are two labor market clearing conditions, one for the skilled workers,

$$L_t^{\text{sh}} = L_t^{\text{sh}} l_t^{\text{sh}} = n_t x_t(j)^{\text{new}}, \quad (3a)$$

and another one for unskilled workers,

$$L_t^{\text{uh}} = L_t^{\text{uh}} l_t^{\text{uh}} = (1 - n_t)x_t(j)^{\text{old}}. \quad (3b)$$

The superscript 'h' indicates the total amount of hours of labor, and obviously $L = L_t^{\text{sh}} + L_t^{\text{uh}}$.

3.4 Final Sector

There is a unique final good, and a continuum of intermediate goods $x_t(j)$ compatible with the GPT in use. The final product is made by 'labor' according to a constant-returns technology and a constant elasticity of substitution (CES):

$$Y_t = \left(\int_0^1 (A(j)x_t(j))^\alpha dj \right)^{\frac{1}{\alpha}}, \quad (4)$$

with $\alpha > 0$. $A(j)$ is the productivity level of the firms, where $A(j) = 1$ if the firms are applying the old GPT and $A(j) = \gamma > 1$ if the firms have successfully innovated. Consequently, $A(j)$ denotes the skill bias of the technology, and we assume that $A(j)$ is exogenously given. In addition, $x_t(j)$ is the flow of

intermediate goods currently used in the production of the final good, using a one-for-one technology. Hence, $x_t(j)$ also denotes the labor demand flow.

Let P_t and $P_t(j)$ denote the price of the final good and the prices of intermediate goods respectively. The final-goods market is perfectly competitive, and the final producers take prices as given, choosing $x_t(j)$ from the maximization of their profits Π_t at time t :

$$\max_{x_t(j)} \Pi_t = \max_{x_t(j)} \left\{ P_t Y_t - \int_0^1 P_t(j) x_t(j) dj \right\}.$$

The final good is produced with either the old GPT ($x_t^{\text{old}}(j)$) or the new GPT ($x_t^{\text{new}}(j)$). The higher quality of these different products is reflected in productivity improvements of the final good. Since the production of the final good requires intermediate goods, the total cost is simply the quantity of intermediate goods used multiplied by their respective prices.

From the (necessary) first-order conditions (FOCs) of this unconstrained optimization problem, we obtain the demands for the intermediate goods using either the new or old GPT. The demand for intermediate goods produced with new technology is

$$x_t^{\text{new}}(j) = \gamma^{\frac{\alpha}{1-\alpha}} \left(\frac{P_t^{\text{new}}(j)}{P_t} \right)^{\frac{1}{\alpha-1}} Y_t. \quad (5a)$$

The demand for intermediate goods produced with old technology is given by

$$x_t^{\text{old}}(j) = \left(\frac{P_t^{\text{old}}(j)}{P_t} \right)^{\frac{1}{\alpha-1}} Y_t. \quad (5b)$$

Intermediate goods produced using new technology increase the efficiency of the manufacture of the end product. Similarly, the demand for $x_t^{\text{new}}(j)$ is a positive function of the technological productivity level. The increase in the demand for intermediate goods using the new GPT is greater than the increase in the demand for goods using the old GPT; the higher the substitutability between intermediate goods ($0 < \alpha < 1$), the higher the demand for goods produced with new technology.

3.5 Intermediate Sector

The intermediate sector uses only labor in its production. The intermediate products can be manufactured using either new or old technology. The production function of the intermediate sector is linear in the labor for a one-for-one technology. Firms using the new GPT have skilled workers to produce $x_t^{\text{new}}(j)$, whereas firms producing $x_t^{\text{old}}(j)$ can employ both types of workers. In the case that the market is not segmented (yet), the skilled workers also offer labor to firms creating $x_t^{\text{old}}(j)$, so that the production function retains its linearity and the one-for-one technology property. Hence, the demands $x_t^{\text{new}}(j)$ plus $x_t^{\text{old}}(j)$ equal the demands for L_t^{sh} plus L_t^{uh} . Once the labor market is segmented, the skilled workforce is completely hired by firms using the new GPT. In that case, production function is given by

$$n_t x_t(j)^{\text{new}} = L_t^{\text{sh}}. \quad (6a)$$

Analogously, the unskilled workers can only work for firms applying the old GPT. Therefore, the corresponding production function reads

$$(1 - n_t)x_t(j)^{\text{old}} = L_t^{\text{uh}}. \quad (6b)$$

These intermediate producers are monopolists, which means that they dictate the price levels. Thus, producers maximize profits, and labor is the only cost:

$$\max_{P_t(j)} \left\{ P_t(j)x_t(j) - w_t(i)L_t^{\text{h}}(i) \right\}.$$

With equations (6) we can modify the above maximization for firms using the new GPT

$$\max_{P_t^{\text{new}}(j)} \left\{ (P_t^{\text{new}}(j) - w_t^{\text{s}}(i)) \gamma^{\frac{\alpha}{1-\alpha}} \left(\frac{P_t^{\text{new}}(j)}{P_t} \right)^{\frac{1}{\alpha-1}} Y_t \right\},$$

and for firms using the old GPT

$$\max_{P_t^{\text{old}}(j)} \left\{ (P_t^{\text{old}}(j) - w_t^{\text{u}}(i)) \left(\frac{P_t^{\text{old}}(j)}{P_t} \right)^{\frac{1}{\alpha-1}} Y_t \right\},$$

From the related FOCs, we obtain the relative prices for the firms using the new GPT ($p_t^{\text{new}}(j) = P_t^{\text{new}}(j)/P_t$),

$$p_t(j)^{\text{new}} = \frac{w_t^{\text{s}}(i)}{\alpha}, \quad (7a)$$

and likewise for the firms using the old GPT ($p_t^{\text{old}}(j) = P_t^{\text{old}}(j)/P_t$),

$$p_t(j)^{\text{old}} = \frac{w_t^{\text{u}}(i)}{\alpha}. \quad (7b)$$

When the elasticity of substitution between intermediate goods is higher, the mark-up becomes smaller. Suppose, for instance, that skilled employees are unable to find a job in the sectors using the new GPT, or that the wages paid by these sectors are still lower in comparison with the wages paid by sectors utilizing the old GPT. Rationally, these professionals accept to work at firms in sectors using the old GPT. Therefore, at the outset of the technology diffusion, the labor market is not segmented, as assumed previously, so that $w_t^{\text{s}} = w_t^{\text{u}} = w_t$, and thus $p_t^{\text{new}} = p_t^{\text{old}}$.

The final good is numeraire, so $P_t = 1$. Substituting equations (7) into equations (5a) and (5b), we obtain the demand for intermediate goods and labor as functions of the wages w_t and the output Y_t :

$$x_t(j)^{\text{new}} = \left(\frac{w_t^{\text{s}}(i)}{\alpha \gamma^{\alpha}} \right)^{\frac{1}{\alpha-1}} Y_t, \quad (8a)$$

$$x_t(j)^{\text{old}} = \left(\frac{w_t^{\text{u}}(i)}{\alpha} \right)^{\frac{1}{\alpha-1}} Y_t. \quad (8b)$$

for firms using the new and old GPT respectively. Equations (8) state that the higher the economy output, the technology productivity γ , and the substitutability between intermediate goods, the higher the demand; the higher the labor cost, the smaller the demand for intermediate goods.

The final good serves as numeraire. Substituting the relative prices into the profit function, we find the profits for both types of firms:

$$\Pi_t^{\text{new}}(j) = \frac{1-\alpha}{\alpha} w_t^s(i) x_t^{\text{new}}(j), \quad (9a)$$

$$\Pi_t^{\text{old}}(j) = \frac{1-\alpha}{\alpha} w_t^u(i) x_t^{\text{old}}(j). \quad (9b)$$

The total amount of profits is of course the sum of profits for both sectors, that is $\Pi_t^{\text{total}} = (1 - n_t) \Pi_t^{\text{old}}(j) + n_t \Pi_t^{\text{new}}(j)$.

From equations (8) we find the skill premium

$$SP = \frac{w_t^s(i)}{w_t^u(i)} = \gamma^\alpha \left(\frac{x_t^{\text{new}}(j)}{x_t^{\text{old}}(j)} \right)^{\alpha-1}. \quad (10)$$

As discussed by [Acemoglu \(2002\)](#), in the case of perfect substitutability among intermediate goods ($\alpha = 1$), the production function is linear and the skill premium translates to an increase in the productivity of the new GPT. In the case of $\alpha = 0$, the production function is the well-known Cobb–Douglas one, and the technology is not biased towards either labor factor; the SP is linear in the relative demand $x_t^{\text{new}}/x_t^{\text{old}}$. If $\alpha \rightarrow \infty$, then the production function reduces to the Leontief production function, and there is no substitutability between intermediate goods, so that the SP tends to infinity. In instances of imperfect substitutability, we observe two special cases. If $\alpha > 1$, the intermediate goods are gross complements, so that both the skill premium and the relative demand increase with the degree of substitutability. If $0 < \alpha < 1$, the intermediate goods are gross substitutes, for which the skill premium decreases.

Substituting the labor demand (8) into the respective labor market clearing conditions (3), we find that

$$w_t^s(i) = \alpha \gamma^\alpha \left(\frac{n_t Y_t}{m_t l_t^s(i)} \right)^{1-\alpha}, \quad (11a)$$

$$w_t^u(i) = \alpha \left(\frac{(1 - n_t) Y_t}{(1 - m_t) l_t^u(i)} \right)^{1-\alpha}, \quad (11b)$$

in the case of segmented markets, and

$$w_t(i) = \alpha \left[\left(1 + n_t \left(\gamma^{\frac{1}{1-\alpha}} - 1 \right) \right) \frac{Y_t}{l_t(i)} \right]^{1-\alpha}, \quad (12)$$

in the case of unsegmented markets. We see that the wages of skilled and unskilled workers are determined by i) the productivity of the respective technologies, ii) the degree of substitutability between the goods produced, iii) the technology diffusion, iv) the economic output, v) the education level of the society, and vi) individual labor decisions.

We can also rewrite the SP in terms of the labor supply:

$$\frac{w_t^s(i)}{w_t^u(i)} = \gamma^\alpha \left(\frac{n_t}{1 - n_t} \frac{1 - m_t}{m_t} \frac{l_t^u(i)}{l_t^s(i)} \right)^{1-\alpha}. \quad (13)$$

As one last step in the basic exposé of our model, we substitute the labor demand and wages (equations

(11) or (12)) into the production function (4) to obtain for segmented markets that

$$Y_t = \left[(1 - n_t)^{(1-\alpha)} \left((1 - m_t) l_t^u(i) \right)^\alpha + \gamma^\alpha n_t^{(1-\alpha)} \left(m_t l_t^s(i) \right)^\alpha \right]^{\frac{1}{\alpha}}, \quad (14)$$

and for unsegmented markets that

$$Y_t = \left[\left(1 - n_t \left(1 - \gamma^{\frac{\alpha^2}{\alpha-1}} \right) \right) \left(\frac{1 + n_t \left(\gamma^{\frac{1}{1-\alpha}} - 1 \right)}{l_t(i)} \right)^{-\alpha} \right]^{\frac{1}{\alpha}}. \quad (15)$$

These equations give the output of the economy as functions of the labor rather than the intermediate goods. Henceforth, we simplify the notation and suppress the arguments i and j .

4 Households Optimization

The individuals from skilled households are already skilled at the beginning ($t = 0$), and they remain so until the end ($t = T$). Unskilled individuals are allowed to invest in education in order to climb up the skills ladder and become part of the skilled subpopulation. Thus, households may raise their human-capital stock, that is, increase the proportion of skilled members tomorrow by allocating a fraction of their time to education or schooling today. Households have preferences over their own consumption and leisure; the maximum number of working hours, that is the length of one day, is normalized to unity. Since all households maximize their expected lifetime utility rather than their instantaneous utility, they do not send all their members to school at the same time. The education productivity is given by

$$p_{t+1} = a l_t^e, \quad (16)$$

with $0 < a < 1$, which we interpret as the probability of an unskilled household becoming skilled at the next period. Hence, they receive income from both types of labor.

The skilled individuals allocate their time between leisure ($1 - l_t^s$) and labor (l_t^s), whereas unskilled agents can now allocate their time between labor (l_t^u), education (l_t^e), and leisure ($1 - l_t^u - l_t^e$). Identical economic agents maximize their expected lifetime, subject to their budget constraints.

The subpopulations of either type are, therefore, endogenously determined as a result of schooling and/or training investments. The subpopulations of skilled and unskilled individuals are fractions m_t and $(1 - m_t)$ of the total population respectively:

$$L_t^s = m_t L, \quad (17a)$$

$$L_t^u = (1 - m_t) L. \quad (17b)$$

The number of skilled workers increases, and consequently the number of unskilled workers decreases over time; the population of skilled workers evolves according to the following equation:

$$m_{t+1} = m_t + (1 - m_t) p_{t+1}, \quad (18)$$

where $0 < p < 1$ is the probability of becoming skilled, which is given by the education production function.

Given that the inequality analysed in this paper concerns the wage differential between skilled and unskilled workers, we postulate that

Assumption 2. *Profits from intermediate sectors are distributed equally between the skilled and unskilled agents.*

This assumption does not change the main results of our model and allows us to concentrate on the wage differential.

Households are forward-looking, which means that they maximize their total *expected* (i.e. current and future) utility. Therefore, we represent the households optimization problem by the Bellman equation, which can be solved by backward induction. The ‘state variables’ are the probabilities m_t and n_t , and the ‘controls’ are the consumptions and labor of both the skilled and unskilled agents, and the education, as described below. Despite skilled workers face a static problem we decided to set it up recursively to facilitate the interpretation of the unskilled optimization problem, once it depends on the value function of skilled individuals⁷.

Problem 1. *For the skilled agents, the dynamic program reads*

$$V_t^s(m_t, n_t) = \max_{c_t^s, l_t^s} \{u(c_t^s, 1 - l_t^s) + \beta V_{t+1}^s((m_{t+1}, n_{t+1}))\},$$

$$\text{s.t. } c_t^s \leq w_t^s l_{s,t} + \Pi_t^{\text{total}},$$

and for the unskilled agents

$$V_t^u(m_t, n_t) = \max_{c_t^u, l_t^u, l_t^e} \{u(c_t^u, 1 - l_t^u - l_t^e) + \beta((1 - al_t^e) V_{t+1}^u(m_{t+1}, n_{t+1}) + al_t^e V_{t+1}^s(m_{t+1}, n_{t+1}))\},$$

$$\text{s.t. } c_t^u \leq w_t^u l_t^u + \Pi_t^{\text{total}}.$$

From Problem 1 we can easily deduce that

$$\frac{\partial u}{\partial c_t^I} - \lambda_t^I = 0, \tag{19a}$$

$$\frac{\partial u}{\partial l_t^I} + \lambda_t^I w_t^I = 0, \tag{19b}$$

$$\frac{\partial u}{\partial l_t^e} + a\beta (V^s(m_{t+1}, n_{t+1}) - V^u(m_{t+1}, n_{t+1})) = 0, \tag{19c}$$

with $I = \{u, s\}$. Equations (19) express that the skill premium is equal to the ratio of the marginal rate of substitution (MRS) of skilled and unskilled households.

4.1 Recursive Equilibrium

The state variables have to be defined in aggregate terms because the households’ decisions depend not only on their past but also on future decisions. Therefore, the households not only require knowledge

⁷The value function for unskilled households (V_t^u) depends on V_{t+1}^s . Therefore, when we solve the model numerically, we solve a system of two equations which can be expressed matrixially as

$$\begin{bmatrix} V_t^s \\ V_t^u \end{bmatrix} = \max \begin{bmatrix} u_t^s \\ u_t^u \end{bmatrix} + \beta \begin{bmatrix} 1 & 0 \\ p_{t+1} & (1 - p_{t+1}) \end{bmatrix} \begin{bmatrix} V_{t+1}^s \\ V_{t+1}^u \end{bmatrix}.$$

This approach allow us to get the value of V^s directly, and we can easily substitute V^s into the value function of unskilled individuals (V^u).

of their own states, but of the economy as a whole.

Definition 1. *The recursive equilibrium for the economy is a set of values for the skilled households $\{V_t^s, c_t^s, l_t^s\}_{t=0}^T$ and the unskilled households, $\{V_t^u, V_t^s, c_t^u, c_t^s, l_t^u, l_t^s, l_t^e\}_{t=0}^T$, a set of production plans for the firms $\{x_t^{\text{new}}, x_t^{\text{old}}\}_{t=0}^T$, and a set of prices $\{w_t^s, w_t^u\}_{t=0}^T$, such that these solve Problem 1 under the aforementioned assumptions.*

So far, we have not specified the utility function, that is the objective function in Problem 1. We take a non-separable, non-isoelastic utility function with preferences according to the Cobb–Douglas form (see e.g. Chari and Christiano, 1994). For the skilled households the utility function is simply the familiar Cobb–Douglas utility function (20), but for the unskilled households we take $u(c_t^u, 1 - l_t^u - l_t^e)$, as their leisure has been reduced to $1 - l_t^u - l_t^e$, because of the time spent on education:

$$u(c_t^l, 1 - l_t^l) = \begin{cases} \frac{1}{1-\sigma} \left((c_t^l)^\eta (1 - l_t^l)^{1-\eta} \right)^{1-\sigma}, & \text{for } \sigma \neq 1, \\ \eta \log c_t^l + (1 - \eta) \log (1 - l_t^l), & \text{for } \sigma = 1. \end{cases} \quad (20)$$

Here, $\eta \in (0, 1)$ determines the relative importance of consumption in comparison with leisure, and σ is the risk aversion of consumers. Related to these parameters are the coefficient of relative risk aversion, abbreviated CRRA, $\sigma\eta + 1 - \eta$, and inter-temporal elasticity of substitution in consumption, or IES for short, $1 / (1 - (1 - \sigma)\eta)$.

From the FOCs (19) for the Cobb-Douglas utility function, we see that

$$c_t^u = \frac{\eta}{1 - \eta} w_t^u (1 - l_t^u - l_t^e), \quad (21a)$$

$$c_t^s = \frac{\eta}{1 - \eta} w_t^s (1 - l_t^s), \quad (21b)$$

for all σ .

Similarly, from the FOCs and the budget constraints, we get the supply of working hours and hours spent on schooling activities:

$$l_t^s \geq \eta - (1 - \eta) \frac{\Pi_t^{\text{total}}}{w_t^s}, \quad (22a)$$

$$l_t^u \geq \eta - (1 - \eta) \frac{\Pi_t^{\text{total}}}{w_t^u}, \quad (22b)$$

$$l_t^e \geq \begin{cases} 1 - l_t^u - \frac{1-\eta}{\beta a (V_t^s - V_t^s)} & \sigma = 1, \\ 1 - l_t^u - \frac{1-\eta}{\beta a (V_t^s - V_t^u)} \frac{c_t^u}{u_t^u}, & \sigma \neq 1 \end{cases} \quad (22c)$$

5 Numerical Results

5.1 Calibration

The agents transit from one state (low skill) to another (high skill) by means of discrete endogenous schooling decisions. The two classes of workers are interpreted as high-school-educated (unskilled) and college-educated (skilled) labor.

All parameters have been calibrated to replicate observations from the US economy in the period 1969–1996, in particular the fraction of the higher educated labor force (skills supply) m_t and the skill premium w_t^s/w_t^u . The relative supply of skills is calculated from a sample that includes all workers be-

tween the ages of 18 and 65. It is defined as the ratio of college-equivalent to non-college-equivalent (Autor and Krueger, 1998), using the number of weeks worked as weights. Here, a college-equivalent individual is exactly the same as a college graduate plus one-half the number of workers with some college experience. The skill premium is identified as the college premium, where the skilled laborers are defined in a similar way as in the works of Acemoglu (2002) and Autor and Krueger (1998).

Preference parameters are calibrated to standard values used by Chari and Christiano (1994), Werning (2007), and Conesa et al. (2009). For the logarithmic utility function, we set $\eta = 0.393$, and the annual discount factor $\beta = 0.95$. Alternatively, we take $\sigma = 2$ with $\eta = 0.42$ as a consistency check. For the values of the technology parameters we follow Aghion (1998); the degree of substitutability across intermediate goods $\alpha = 0.8$, and the productivity improvement of the new technology $\gamma = 1.5$.

Several runs of simulations indicate that for a range of different factors ($1.25 \leq \gamma \leq 1.75$), we obtain transition paths that exhibit the same features as the original one. Again, the overall trends do not change if we modify the elasticity to a smaller value of 1.67, which corresponds to $\alpha = 0.4$.

Regarding the productivity of the investment in education, we set $a = 0.65$, which relates directly to the dynamical transition probability p_t , as given by equation (16). We take $m_0 = 0.27$ from the empirical data, and we assume that $n_0 = 0.30$. The latter value is chosen to match the empirical value of the proportion of skilled workers in 1969. The parameters of our model are summarized in the Table 1.

	η	β	γ	α	a
$\sigma = 1$	0.393	0.95	1.5	0.80	0.65
$\sigma = 2$	0.42	0.95	1.5	0.80	0.65

Table 1: Parameter values.

5.2 Transition Dynamics

The economy has two steady states: the initial state, in which there is only the old GPT ($n_0 = 0$), and the final state, in which the new GPT has spread out across the entire economy ($n_T = 1$). Near these equilibria there are undetermined equilibria. In order to compute the recursive equilibrium numerically, we take $n_0 > 0$ initially, also to match empirical evidence, and the final n_T close to unity. We also assume that all individuals live $T = 40$ years and $V_{T+1}^I = 0$, for $I = \{u, s\}$. With that information and an estimate for m_T we can solve the model recursively with the Bellman equation for either category of workers. The assumption that $T < \infty$ is necessary in order to obtain a numerical solution. However, the main results are unaffected by the specific value of T . Numerous numerical simulations suggest that the model retains its characteristic features over extended periods of time. At each period, we are left with a nonlinear programming (NLP) that we solve with an accurate (global) optimization algorithm. So, we restate the multi-objective recursive optimization problem 1 as a nonlinear programming problem, which we achieve by combining the value functions (with equal weights) into one objective function, as described in A.1.

Table 2 summarizes the main results for the economy and in more detail in Figures 2 – 5. In each figure, the left-hand side displays the outcome for the economy with $\sigma = 2$ and the right-hand side the results with $\sigma = 1$.

In 1969 the relative supply of skills in the USA was 27% and, according to our model (cf. Table 2), the skill-biased technology accounted for 93% (1.45) of the total skill premium observed (1.55). Then,

Model quantity	US data			Model ($\sigma = 1$)			Model ($\sigma = 2$)		
	1969	1980	1996	1969	1980	1996	1969	1980	1996
w_t^s/w_t^u	1.55	1.43	1.75	1.45	1.31	1.59	1.45	1.31	1.62
m_t	0.27	0.43	0.57	0.27	0.38	0.66	0.27	0.36	0.63

Table 2: Comparison of the model results with the data for the US.

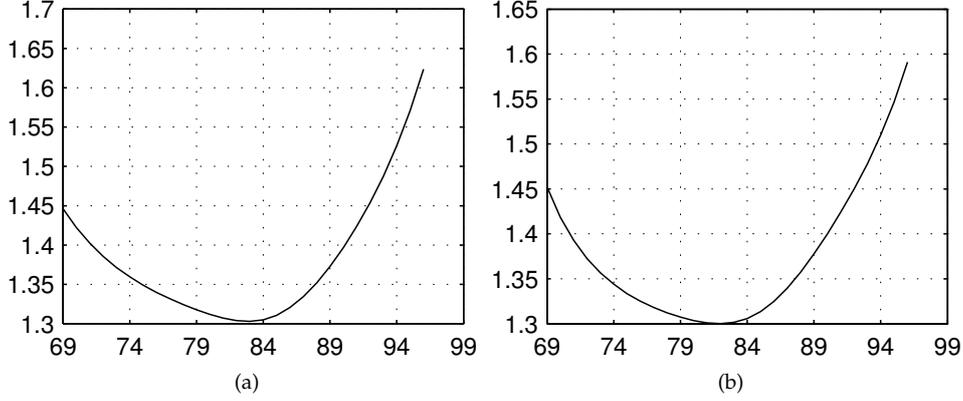


Figure 2: Skill Premium for the CES and logarithmic utility functions.

suddenly, the technology began to diffuse through all sectors and firms. Economic agents (households) foresaw its effects and took their decisions of consumption, schooling and labor supply during this economic transition into account.

The model exhibits the observed transitional dynamic path for both skill premium and skills supply (see Table 2, Figures 2 and 6). The US data indicates a slowdown in the SP between 1969 and 1979, which is also retrodicted by our model. However, our results suggest that the slowdown should have lasted approximately three years longer. This difference is related to the fact that n_t is still exogenous in our model; we have assumed a constant arrival rate for the template of the new GPT. The model not only retrodicts the SP slowdown but also the acceleration in the SP after 1979. For example, during the US economic slowdown (1969–1980), the skill premium dropped 12 percentage points while the skills supply increased 16 percentage points. Our model (for $\sigma = 1$) reports a fall of 14 percentage points for the skill premium, and a rise of 11 percentage points for the skills supply. For 1980 we predict in retrospect that the skill premium and the relative skills supply are 1.31% and 0.38% respectively. So, a diffusion rate of 43% is able to account for 92% and 82% of the real values of the skill premium and skills supply. During the US economic expansion (1980–1996), the skill premium and skills supply increased 22 and 14 percentage points respectively. In our model we observe a rise of 28 percentage points for both the skill premium and the skills supply. Nevertheless, we can account for 90% of the real value of the skill premium in 1996, overestimating the skills supply by 15%, when 83% of the firms in the economy are using new technology.

Apart from the income inequality, there is a consumption inequality because of the technology diffusion, as one can see in Figure 3. Our results display both an intra- and inter-temporal consumption inequality as a direct consequence of the income inequality between educational groups. This side effect has not been researched in the partial-equilibrium analysis conducted by [Aghion \(2002\)](#); [Acemoglu \(2002\)](#); [Krusell et al. \(2000\)](#); in the context of welfare and redistributive policies, the consumption inequality is actually more valuable, so that the link between the technology diffusion and the consumption inequality is of great importance.

Note that, although the overall consumption (90/10 ratio) does not reflect this increase in inequality, one can observe a decline⁸ in an empirical decomposition of the ‘within’ education groups consumption inequality and a sharp increase in the ‘between’ education groups consumption inequality, which is not so different in magnitude from the income inequality (see e.g., [Attanazio, 1999](#); [Krueger and Perri, 2006](#)). This suggests that changes in the relative wages between education groups are mirrored in the changes in the consumption of these groups.

The supply of skilled (unskilled) labor increases (decreases) over time at a non-constant rate (Figure 4) due to variable changes in the skill premium and profits. Furthermore, in roughly the first fifteen years of the new GPT diffusion, the unskilled laborers work more than skilled ones, whereas in the last twenty-five years or so the situation is reversed, supporting the empirical evidence as in [Erosa et al. \(2009\)](#) and [Heathcote et al. \(2010\)](#). This means that the skilled workers always consume more than their unskilled co-workers, as shown in Figure 3. The income effect is visible in Figures 3 and 4, where in the initial phase the wages w_t^s decrease and the amount of labor l_t^s increases. Subsequently, the skilled wages follow an upward trend, which is the substitution effect while unskilled wages follow a downward trend (Figure 6), as shown by empirical evidence.

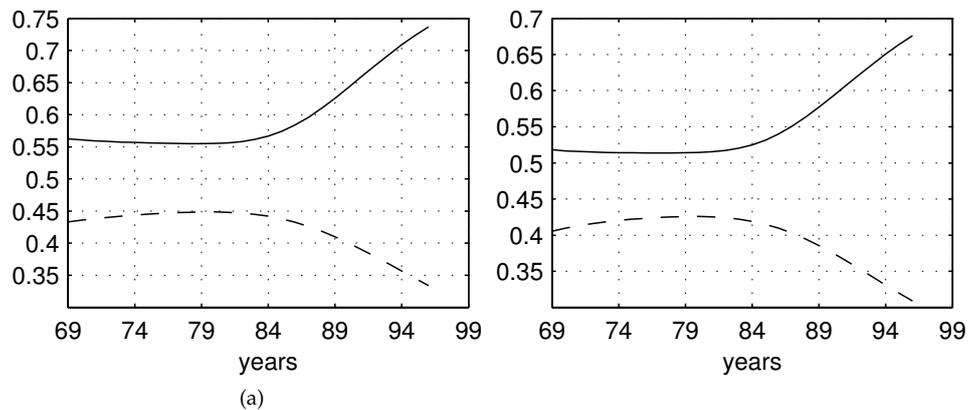


Figure 3: Consumption for the CES and logarithmic utility functions; skilled (solid) and unskilled (dashed).

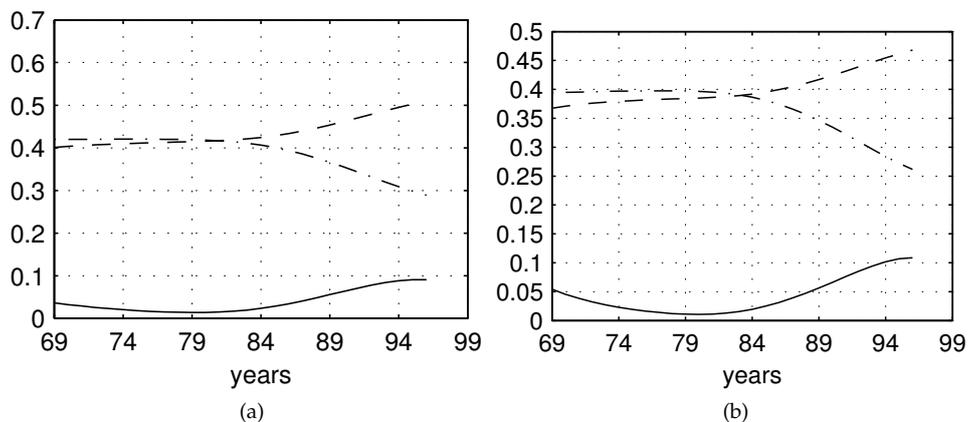


Figure 4: Labor supply for the CES and logarithmic utility functions; education (solid), skilled (dotted) and unskilled (dashed).

⁸[Attanazio \(1999\)](#) suggest that this decline of the ‘within’ consumption inequality is due to improved intra-group risk sharing, while the rise in the ‘between’ consumption inequality constitutes a falsification of the hypothesis of the between-groups consumption inequality.

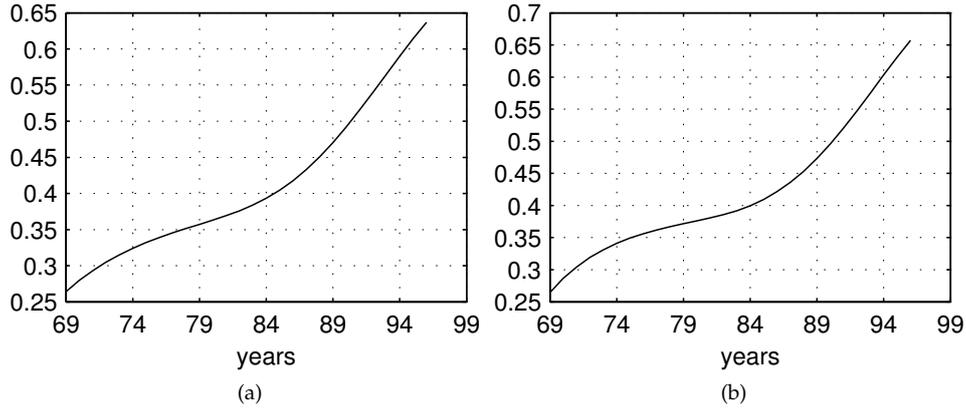


Figure 5: Skills supply for the CES and logarithmic utility functions.

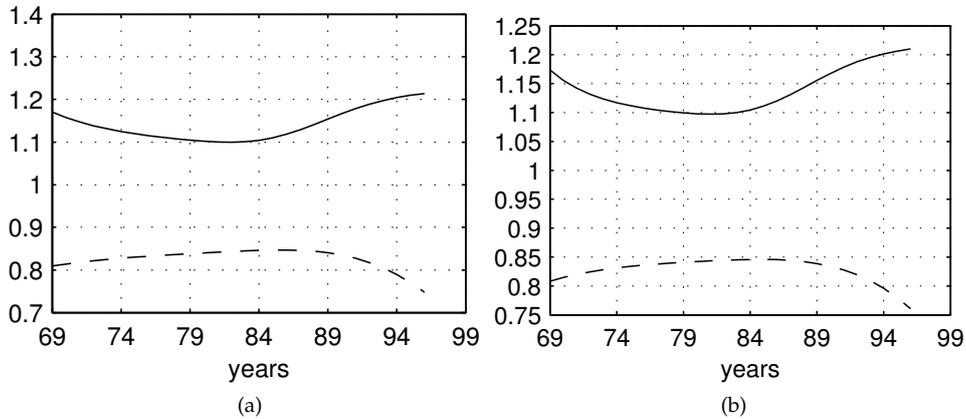


Figure 6: Wages for the CES and logarithmic utility functions; skilled (solid) and unskilled (dashed).

Similar behavior is observed in the results for unskilled personnel; since skilled and unskilled individuals have opposite reactions to changes in the wages (Figure 6), the resulting higher wage inequality exacerbates the consumption inequality (Figure 3). Regarding the investment in education (Figure 4), college-educated people work more than their high-school-educated colleagues, not only because of substitution/income effects, but also because their co-workers sacrifice their spare time to study.

The investment in education (Figure 4) and the human-capital accumulation (Figure 5) tend to be proportional to the acceleration of the skill premium, which in turn means that they are proportional to the technology diffusion rate. Indeed, as soon as the technology diffusion rate decreases, and similarly the skill premium, we observe a slowdown in the investment in education. Note that the slowdown of technology diffusion is reflected in the investment in education. This feature is also found in the real data. Goldin and Katz (2007) show that beyond the 90s, the growth rate of college-educated individuals has decreased.

6 Conclusion

In this paper we have studied the supply side effects of a skill-biased technology diffusion, defining a general-equilibrium model with leisure–labor decisions and human-capital accumulation. We analyze how households change their optimal labor, consumption and investments in education decisions over

the technology cycle under imperfect credit market assumption.

We show that in a general-equilibrium framework the concept of the skill-biased technology change, in conjunction with technology diffusion, explains why there was an increase in the proportion of skilled individuals and, at the same time, why the SP decreased in the United States between 1969 and 1979, and increased between 1980 and 1996. This is an important feature reproduced by our model. Neither pure technology progress nor diffusion models provide an explanation for the evolution of the skills supply (Acemoglu, 2002; Aghion, 2002). Acemoglu (1998), in a first attempt to explain the skills supply, focus on the analysis of the BGP and assumes a constant proportion of skills supply and the SP. Instead, we have provided an explanation for economies in transition and allow for changes in the growth rate of the skills supply and SP as observed in the USA from 1969 to 1996.

Our model demonstrates that the labor supply of skilled relative to unskilled workers increases as a result of the dominance of the income effect, in spite of skilled laborers initially working less. Because of the wage differential the higher-educated workforce consumes more and more in contrast to its lower-educated counterpart. Our model is able to match both the overall trends and the finer structures in the data for the USA, showing that the individuals' decisions to invest in education are indeed a consequence of the diffusion (as opposed to the innovation) of an SBTC. The model reproduces the decline in unskilled wages too.

We also show that the higher the proportion of skills at the beginning of the diffusion process, the smaller the SP and the faster we reach a fully educated work force. Thus, every time a skill-biased technology change comes along the technology cycle re-starts. This suggests that economies that base their economic growth on the diffusion of new SBTCs face a divergent economic growth process.

Although our results are consistent with empirical data, discrepancies have been noticed. In order to emulate the data more accurately, models that include physical capital, externalities and real data for the SBTC diffusion are required. These are topics for future investigations.

A Appendix

A.1 Computational Method

Problem 1 is a finite-horizon discrete-time dynamic optimization problem. At each phase (period) $t = 0, \dots, T$ with $T \geq 0$, the dynamic variables m_{t+1} and n_{t+1} , and the value functions V_{t+1}^I are (assumed to be) known. Since all agents live T periods, we have that $V_{T+1}^I = 0$. Initially, we guess m_T , and refine its value after each optimization according to the computed value m_0 . In that way we obtain a sequence of $(T + 1)$ nonlinear programming problems (NLPs), which can be solved by backward induction, as is commonly done. By virtue of the principle of optimality, the solution found with the dynamic programming approach is the global optimum of the original problem.

Once the model has been calibrated, and the algorithms have been initialized, the optimization sequence starts at phase $t = T$. A genetic algorithm (GA) attempts to find the global optimum and once it has found a candidate solution to the NLP, it passes that solution as an initial guess to an SQP algorithm, which in turn refines the solution and returns the global optimum of the NLP. It might be possible that the GA is unable to close in on the optimum due to for instance an infeasible initial population or too stringent tolerances, so we have allowed for the possibility to automatically restart the GA several times with different linearly feasible yet random initial populations. The solution found in the

segmented market structure determines whether the algorithm stores and sets all values for the subsequent phase immediately ($w_t^u < w_t^s$), or proceeds with the optimization in the unsegmented market structure first ($w_t^u \geq w_t^s$). The optimization sequence is repeated from phase $t = T$ to $t = 0$, after which the dynamic program has been solved.

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